AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

43. Proposed by HENRY HEATON, M. Sc., Atlantic, Iowa.

In a circle whose radius is a, chords are drawn through a point distant b from the center. What is the average length of such chords, (1), if a chord is drawn from every point of the circumference, and (2), if they are drawn through the point at equal angular intervals?

II. Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas.

Let $AC=CF=a$, $CD=b$, $\angle FDA=\theta$, $\angle FCA=\varphi$, $\tan\theta=m$.

Then the equation to DF is, $y=mx+mb$ ....(1).

The equation to the circle is, $x^2+y^2=a^2$ ....(2).

From (1) and (2) we easily get $EF=2\sqrt{a^2-\dfrac{m^2b^2}{1+m^2}}$,

$\therefore\ EF=2\sqrt{a^2-b^2\sin^2\theta}$. But $\sin\theta=\dfrac{a\sin\varphi}{\sqrt{a^2+b^2+2ab\cos\varphi}}$.

$$\therefore\ EF=\frac{2(a^2+ab\cos\varphi)}{\sqrt{a^2+b^2+2ab\cos\varphi}}=\frac{2(a^2+ab-2ab\sin^2\frac{1}{2}\varphi)}{\sqrt{(a+b)^2-4ab\sin^2\frac{1}{2}\varphi}}.$$

The limits of φ for $b>a$, are 0 and $\frac{1}{2}\pi+\sin^{-1}(a/b)=2\beta$.

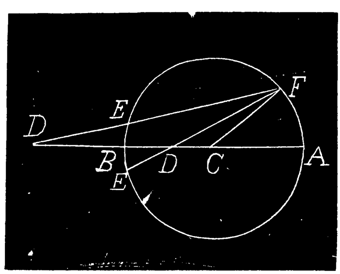
The limits of φ for $b<a$, are 0 and $\frac{1}{2}\pi+\sin^{-1}(b/a)=2\delta$.

The limits of θ for $b>a$, are 0 and $\sin^{-1}(a/b)=\theta'$.

The limits of θ for $b<a$, are 0 and $\frac{1}{2}\pi$.

Let Δ and Δ_1 be the average lengths required.

$$\therefore\ \Delta=\int EFds\Big/\int ds=\int EFd\varphi\Big/\int d\varphi,$$



$$\Delta_1=\int EFd\theta\Big/\int d\theta.$$

I. Let $\dfrac{4ab}{(a+b)^2}=e^2$, and $\frac{1}{2}\varphi=\gamma$.

$\therefore\ d\varphi=2d\gamma$.

Then $EF = \dfrac{2(a^2+ab-2ab\sin^2\gamma)}{(a+b)\sqrt{1-e^2\sin^2\gamma}}$.

$$\therefore\ EF = \frac{2a}{\sqrt{1-e^2\sin^2\gamma}} + \frac{2}{e\sqrt{ab}}\sqrt{1-e^2\sin^2\gamma} - \frac{2}{e\sqrt{ab}\sqrt{1-e^2\sin^2\gamma}}.$$

$$\therefore \int EF d\gamma = \frac{2}{e\sqrt{ab}}\{(ae\sqrt{ab}-1)F(e,\gamma)+E(e,\gamma)\}.$$

$$\therefore\ \Delta = \frac{2}{\beta e\sqrt{ab}}\{(ae\sqrt{ab}-1)F_0^\beta(e,\gamma)+E_0^\beta(e,\gamma)\},\ b>a.$$

$$\Delta = \frac{2}{\delta e\sqrt{ab}}\{(ae\sqrt{ab}-1)F_0^\delta(e,\gamma)+E_0^\delta(e,\gamma)\},\ b<a.$$

II. $\Delta_1 = 2\int_0^{\theta'}\sqrt{a^2-b^2\sin^2\theta}\,d\theta \Big/ \int_0^{\theta'} d\theta = \dfrac{2}{\theta'}\int_0^{\theta'}\sqrt{a^2-b^2\sin^2\theta d\theta}$.

Let $\theta = \frac{1}{2}\pi+\lambda$. $\therefore\ \theta'-\frac{1}{2}\pi=\lambda$ to $-\frac{1}{2}\pi=\lambda$.

$$\therefore\ \Delta_1 = \frac{2}{\theta'}\int_{-\frac{1}{2}\pi}^{\theta'-\frac{1}{2}\pi}\sqrt{a^2-b^2\cos^2\lambda}\,d\lambda = \frac{2}{\theta'}\int_{-\frac{1}{2}\pi}^{\theta'-\frac{1}{2}\pi}\sqrt{b^2\sin^2\lambda-(b^2-a^2)}d\lambda$$

$$= \frac{2\sqrt{(b^2-a^2)}}{\theta'}\int_{-\frac{1}{2}\pi}^{\theta'-\frac{1}{2}\pi}\sqrt{\frac{b^2}{b^2-a^2}\sin^2\lambda-1}\,d\lambda$$

$$= \frac{2\sqrt{(b^2-a^2)}}{\theta'} H_{-\frac{1}{2}\pi}^{\theta'-\frac{1}{2}\pi}\left(\frac{b}{\sqrt{b^2-a^2}},\lambda\right),\ b>a.$$

$$\Delta_1 = 2\int_0^{\frac{1}{2}\pi}\sqrt{a^2-b^2\sin^2\theta}\,d\theta\Big/\int_0^{\frac{1}{2}\pi}d\theta = \frac{4a}{\pi}E_0^{\frac{1}{2}\pi}\left(\frac{b}{a},\ \theta\right),\ b<a.$$

45. Proposed by J. C. WILLIAMS, Boston, Massachusetts.

At the end of the fifth inning the base ball score stands 7 to 9. What is the probability of winning for either team ?

Solution by J. SCHEFFER, A. M., Hagerstown, Maryland.

From the stated score we are able to estimate the respective skill of the two teams, and their respective probabilities of winning the game.

The respective probabilities are $\frac{7}{16}$ and $\frac{9}{16}$. We have now to find the probabilities of either team winning *at least* 3 games out of 4, granting, of course, 9